KVL in left loop: \( i_X = \frac{V_S}{0.1 \times 10^3 + 0.4 \times 10^3} = 2 J_S \, \text{mA} \)

KVL in right loop: \( i_0 = \frac{-r \cdot i_X}{0.5 \times 10^3 + 2 \times 10^3} = \frac{-5 \times 10^3 i_X}{2.5 \times 10^3} = -2 i_X \)

Voltage division:

\[
\begin{align*}
J_0 &= \frac{-2 \times 10^3 (R + i_X)}{0.5 \times 10^3 + 2 \times 10^3} = \frac{-2 \times 10^3 (5 \times 10^3 i_X)}{2.5 \times 10^3} = -4 \times 10^3 i_X \\
\frac{V_0}{V_S} &= -4 \times 10^3 i_X \\
\frac{i_0}{i_X} &= -2 i_X
\end{align*}
\]
We can use a supermesh in the above circuit as:

\[-V_S + R_S i_A + R_0 i_B = 0 \quad (\text{I})\]

Also, we have:

\[
\begin{align*}
    i_B - i_A &= g_V i_X \\
    V_X &= -R_S i_A
\end{align*}
\]  \(\Rightarrow i_A = \frac{i_B}{1-gR_S} \quad (\text{II})\)

\[\begin{align*}
(\text{I}) \quad \text{and} \quad (\text{II}) &\Rightarrow -V_S + R_S \frac{i_B}{1-gR_S} + R_0 i_B = 0 \\
\Rightarrow i_B &= \frac{V_S}{R_S \left(\frac{1}{1-gR_S} + R_0\right)}
\end{align*}\]

\[\begin{align*}
V_D &= R_0 i_B = R_0 \frac{V_S}{R_S \left(\frac{1}{1-gR_S} + R_0\right)}
\end{align*}\]

\[\begin{align*}
\frac{V_D}{V_S} &= \frac{R_0}{R_0 + \frac{R_S}{1-gR_S}} = \frac{R_0 (1-gR_S)}{R_0 + R_S - gR_0 R_S}
\end{align*}\]
Based on the i-v relationships of the ideal model of the Op Amp, we have:

\[
\begin{align*}
    V_P &= V_N \\
    I_P &= I_N = 0
\end{align*}
\]

\[\Rightarrow V_P = V_N = 0\]

**KCL at A:**
\[
\frac{0 - V_S}{33 \times 10^3} + \frac{0 - V_O}{330 \times 10^3} = 0 \Rightarrow \frac{V_O}{V_S} = -10
\]

(b)

Again we have,
\[V_P = V_N \Rightarrow V_N = V_P = V_S\]

\[I_P = I_N = 0\]

**KCL at B:**
\[
\frac{V_S}{33 \times 10^3} + \frac{V_S - V_O}{330 \times 10^3} = 0 \Rightarrow \frac{V_O}{V_S} = 11
\]
The variable resistor is denoted by \( R_v \) which is in the range of: \( 0 \leq R_v \leq 100 \, \text{k}\Omega \)

The above circuit is an inverting amplifier with a gain of:

\[
K = \frac{V_o}{V_s} = -\frac{100 \times 10^3 + R_v}{2 \times 10^3}
\]

\( 0 \leq R_v \leq 100 \, \text{k}\Omega \), thus the gain is in the range of:

\[-100 \leq K = \frac{V_o}{V_s} \leq -50\]
a) \( i_P = i_N = 0 \Rightarrow V_P = V_S \)
Also \( V_P = V_N \Rightarrow V_N = V_S \)

KCL at (A):
\[
\frac{V_N}{10 \times 10^3} = \frac{V_D - V_N}{150 \times 10^3} \Rightarrow \frac{V_N}{V_S} = 16
\]

b) \( V_S = 1 \text{V} \Rightarrow V_D = 16 \times 1 = 16 \text{V} \)

\( V_S = 3 \text{V} \Rightarrow V_D = 16 \times 3 = 48 \text{V} \)

In this case, the Op Amp becomes saturated, thus
\( V_D = +24 \text{V} \) (coming from \( V_{cc} = \pm 24 \text{V} \))
Amplification by a factor of 1 ⇒ connect switch 3
because \( V_p = V_N \)
\( V_p = V_S \Rightarrow V_0 = V_S = 2V \)
\( V_N = V_0 \)

Amplification by a factor of 5 ⇒ connect switch 2
because in this case, the circuit acts like a noninverting amplifier with a gain of \( K = \frac{V_0}{V_S} = \frac{40 \times 10^3 + 10 \times 10^3}{10 \times 10^3} = 5 \)
\( V_0 = 5 \times V_S = 10V \)

Amplification by a factor of 10 ⇒ connect switch 1
because again in this case, the circuit acts like a noninverting amplifier with a gain of \( K = \frac{V_0}{V_S} = \frac{90 \times 10^3 + 10 \times 10^3}{10 \times 10^3} = 10 \)
\( V_0 = 10 \times V_S = 20V \)

Due to \( V_{cc} = \pm 15V \), the Op Amp becomes saturated and thus \( V_0 \) can have a maximum value of \( V_0 = 15V \). Therefore, another recommendation to fix the circuit for this case is to increase \( V_{cc} = \pm 15V \) to \( V_{cc} = \pm 20V \).
Using superposition to find $V_0$:

1. First set $V_2 = 0$ (turn off $V_2$);

In above circuit, $V_{P_1} = 0$. Therefore the circuit acts like an inverting amplifier with the result that

$$V_{O_1} = -\frac{33 \times 10^3}{10 \times 10^3} V_1$$

2. Second set $V_1 = 0$ (turn off $V_1$);

The circuit looks like a noninverting amplifier with a voltage divider connected at its input.
Voltage division: \[ V_{P2} = \frac{33 \times 10^3}{10 \times 10^3 + 33 \times 10^3} V_2 \]

Noninverting amplifier: \[ V_{O2} = \frac{33 \times 10^3 + 10 \times 10^3}{10 \times 10^3} V_{P2} \]

\[ V_O = V_{O1} + V_{O2} = -\frac{33}{10} V_1 + \frac{33}{10} V_2 \]

\[ \Rightarrow V_O = \frac{33}{10} (V_2 - V_1) \]

As we can see, the output voltage is proportional to the difference between the two inputs. Thus, such a circuit is called a differential amplifier or subtractor.

In page 195 of the book, we can see the general equation for gain of a subtractor circuit.
Using superposition to find \( V_0 \);

1) First set \( J_{S2} = 0 \):

Circuit looks like a noninverting amplifier with a voltage divider connected at its input.

Voltage division:

\[ V_{P1} = \frac{R_2 J_{S1}}{R_1 + R_2} \]

Noninverting amplifier:

\[ V_{01} = \frac{R_3 + R_4}{R_3} \]

\[ V_{P1} = \frac{R_3 + R_4}{R_3} \cdot \frac{R_2 J_{S1}}{R_1 + R_2} \]

2) Second set \( J_{S1} = 0 \):

The same as case 1:

\[ V_{P2} = \frac{R_1 J_{S2}}{R_1 + R_2} \]

\[ V_{02} = \frac{R_3 + R_4}{R_3} \cdot \frac{R_1 J_{S2}}{R_1 + R_2} \]

\[ V_0 = V_{01} + V_{02} = \left( \frac{R_3 + R_4}{R_3} \right) \left( \frac{1}{R_1 + R_2} \right) \left( \frac{R_2 J_{S1}}{R_3} + \frac{R_1 J_{S2}}{R_1 + R_2} \right) \]