MAE140 Linear Circuits (for non-electrical engs)

Topics covered

Circuit analysis techniques
- Kirchoff’s Laws – KVL, KCL
- Nodal and Mesh Analysis
- Thévenin and Norton Equivalent Circuits
- Resistive circuits, RLC circuits
- Steady-state and dynamic responses
  - Impulse and step responses
  - Laplace transforms
- Sinusoidal steady-state response

Circuit design techniques
- Active circuit elements – dependent sources and operational amplifiers
- Feedback basics
- Signal filtering – if we get time
What do I expect you to know?

Prerequisites

Mathematics 21D or 20D (some are doing this concurrently)
- Solution of sets of linear equations
  \[ Ax=b \] equations for vector \( x \) given matrix \( A \) and vector \( b \)
- Solution of constant coefficient linear ordinary differential equations
  - Laplace transform introduction
  - Initial conditions and forced response

Complex analysis
- Numbers, arithmetic, magnitudes and phases, poles, zeros

Physics 2B
- Behavior of circuit elements: \( R, L \) and \( C \)
- Underlying physical principles
Why should you be excited about MAE140?

Linear circuits are exceptionally well described by simple and powerful mathematics.

The quality of concordance between measured physical behavior and mathematical description is amazing.

This is an analog computer from around 1963. It was used to solve nonlinear ODEs via analog circuitry. It contained: about 20 integrators based on valves or transistors, some nonlinear function blocks using diodes, comparators. This was the state of the art for much physical system simulation. It relies on ideas underpinning MAE140. The tools of MAE140 are immediately useful in design.
Why should you still be excited about MAE140?

The *front and back ends* of your digital cell phone are comprised of analog circuits

This is pretty much true of all *digital* technology

**Why does the stagecoach wheel appear to rotate backwards?**

**Aliasing**

A high frequency masquerading as a low frequency

**Anti-aliasing filters MUST be used in all sampled data systems**

**After MAE140 you will be able to start designing such anti-aliasing filters**

The real thing!
Why should you still be excited about MAE140?

You will learn how these knobs and plugs work:

We will not have time to cover this one:
Why should you still be excited about MAE140?

“... a senior engineer in the Sandia national laboratories responsible for the mechanical safety of nuclear weapons concludes that ‘one simple, dynamo-technology, low voltage switch stood between the United States and a major catastrophe.’

“The bombs fell to earth after a B-52 bomber broke up in mid-air, and one of the devices behaved precisely as a nuclear weapon was designed to behave in warfare: its parachute opened, its trigger mechanisms engaged, and only one low-voltage switch prevented untold carnage.”
Circuit variables (T&R Chap 1)

Charge and Energy

- 1 coulomb (C) = $6.25 \times 10^{18}$ electrons’ charge
- 1 ampere (A) = 1 coulomb/second
  
  Current is a measure of charge passing
  Direction of flow is of positive charge
  This is important for understanding some devices

Voltage/potential difference/electromotive force/tension

Voltage measures the energy gained by a charge

- 1 joule (J)/coulomb = 1 volt (V)
  
  Voltage is measured between two points (potential difference)
  This is akin to gravitational potential and fluid flow or temperature and heat flow
Circuit variables (contd)

Power is the rate of energy change per unit time \[ p = \frac{dw}{dt} \text{ watts (W)} \]

Note the chain rule
\[ p = \left( \frac{dw}{dq} \right) \left( \frac{dq}{dt} \right) = vi \]

The instantaneous electrical power associated with a voltage and a current flow is given by the product \( v \times i \)

Passive sign convention

Power is positive when a device absorbs power

Signs are measured as illustrated

“The current variable must be defined so positive current enters the positive voltage terminal of the device”

Note that \( v(t) \) or \( i(t) \) can be negative

In the illustration the current into the device is the negative of that into the rest of the circuit
An example

The figure at right depicts the circuit inside my flashlight

The devices are the battery and the lamp

A voltmeter reads $v(t)$ is 12V

An ammeter reads $i(t)$ is $-1.5$A

Which device is the battery and which is the lamp? How much power is being consumed by the lamp?

Using the passive sign convention

The power absorbed by device A is $12 \times -1.5 \text{ W} = -18 \text{ W}$

A is the battery and B is the lamp, which consumes 18W
Circuit variables (contd)

Ground

Because all voltages are measured between two points, there is no absolute voltage - it is a potential. In circuits we often refer all voltages of points as being relative to a fixed point or **ground** voltage.

This is depicted via the symbol

\[ \overline{\text{G}} \]

The terms \( v_A \) etc mean the voltage at \( A \) relative to \( G \).

This is like referring heights relative to sea level.

Gravity is also a potential.

Check out the elevator buttons in the Math & App Phys Building!
A circuit is a collection of interconnected electrical devices

For us:

- All electromagnetic interactions in the circuit take place within the devices (This is a lumped-parameter circuit)
- The circuit devices are connected together by wires which are ideal
  - They have the same voltage at both ends instantaneously
  - They propagate current without loss instantaneously
  - They may be stretched arbitrarily without changing properties
- All circuit devices have at least two terminals and are assumed not to accumulate charge – current in equals current out

A node is the junction of terminals of two or more devices

A loop is a closed path formed by tracing through an ordered sequence of nodes without passing through any node more than once
Circuit variables

The important physical circuit variables are currents into and voltages across circuit elements and their associated power consumption or production.

*Circuit analysis* consists of solving for the circuit variables in a given circuit.

*Circuit design* or *synthesis* consists of constructing a circuit whose circuit variables exhibit a specified behavior.

The *topology* of a circuit is the key structure.

It can be stretched and wrapped around without changing circuit variables.

Nodes can be stretched and shrunk to cover many terminals.
Circuit Topology

This is the circuit diagram of an audio amplifier.

It shows the relationship between parts but not their actual physical location.

The ground/earth is likely the chassis or case.

The output transistors BD912 and BD911 will need to be attached to an external heat sink.

The op-amp OPA604 will be board mounted.

The volume control needs to be accessible.
Circuit Topology – high power amplifier with complementary HEXFETS

HEXFET – power MOSFET from International Rectifier
Kirchoff’s Current Law (KCL)

The algebraic sum of currents entering a node is zero at every instant

This is a restatement of the principle of conservation of charge

Alternatively, but not so nice,

The sum of currents entering a node is equal to the sum of currents leaving a node at every instant

KCL provides linear constraints between the currents in a circuit

In a circuit containing a total of \( N \) nodes there are only \( N-1 \) independent KCL connection equations
Write KCL at nodes $A$, $B$, $C$, $D$

Node A: $-i_1 - i_2 = 0$
Node B: $i_2 - i_3 - i_4 = 0$
Node C: $i_4 - i_5 - i_6 = 0$
Node D: $i_1 + i_3 + i_5 + i_6 = 0$

If $i_1 = -1\,\text{mA}$, $i_3 = 0.5\,\text{mA}$, $i_6 = 0.2\,\text{mA}$, find the rest

- $i_2 = 1\,\text{mA}$
- $i_4 = 0.5\,\text{mA}$
- $i_5 = 0.3\,\text{mA}$

Note that we have 4 nodes
But only 3 independent equations
Kirchoff’s Voltage Law (KVL)

The algebraic sum of voltages around a loop is zero at every instant.

This is really a restatement of voltage as a potential, effectively a statement of the principle of conservation of energy.

Example:

Loop 1: \(-v_1 + v_2 + v_3 = 0\)

Loop 2: \(-v_3 + v_4 + v_5 = 0\)

Loop 3: \(-v_1 + v_2 + v_4 + v_5 = 0\)
Circuit Analysis

One way to consider the analysis of a circuit (i.e. the computation of the currents in and voltage across the circuit elements) is through the specification of constraints on these variables

KCL and KVL provide one set of constraints on circuit variables reflecting connection topology

The \( i-v \) properties of the devices themselves provide further constraints

\[ \begin{align*}
N \text{ nodes and } E \text{ circuit elements} \\
N-1 \text{ independent KCL relations} \\
E-N+1 \text{ independent KVL relations (number of independent loops)} \\
E \text{ independent } i-v \text{ characteristics} \\
2E \text{ circuit variables}
\end{align*} \]

If we keep to linear devices then we have linear circuit analysis

\[ 2E \text{ Linear algebraic equations for } 2E \text{ variables} \]
Linear Circuit Elements

Linearity in $v$-$i$ relations

Doubling $v$ doubles $i$ and vice versa $i = \alpha v$

Straight lines in $i$-$v$ plane

Note that $i = \alpha v + \beta$ describes an affine relation

Resistor

$v = R i$ or $i = G v$ Ohm’s Law

$R$ resistance in ohms ($\Omega$), $G$ conductance in siemens (S)

Power $P = i^2 R = v^2 / R = v^2 G$

A resistor always absorbs power – $i$-$v$ line has positive slope

This is an instantaneous linear element

We will look at circuit elements with memory like $C$ and $L$ later in the course
Time out puzzle

We have three lamps – red, green and blue – connected to three switches as in the diagram

The lamps are outside the switch room and we are alone

What single test can we perform by playing with the switches to determine which switch is connected to which light?

I know of two solutions, one of physicists one of engineers
Linear Circuit Elements (contd)

Independent Voltage and Current Source (IVS and ICS)

IVS $v = v_s$ for any $i$

ICS $i = i_s$ for any $v$

These are affine elements rather than linear. But we admit them here.

Open circuit and short circuit – variants of ICS and IVS resp.

Open circuit $i = 0$ for any $v$  \quad \text{Same as } R = \infty

Short circuit $v = 0$ for any $i$  \quad \text{Same as } R = 0

These are truly linear since the curves pass through $(0,0)$

Switches – either o.c. or s.c.
Example 2-10 (T&R, 5th ed, p. 31)

Write a complete set of equations for each element

Write a complete set of connection equations

Solve these equations for all currents and voltages
Example 2-10 (T&R, 5th ed, p. 31)

Write a complete set of equations for each element

\[ v_A = 30V; \]
\[ v_1 = 100i_1; \]
\[ v_2 = 200i_2; \]
\[ v_3 = 300i_3; \]

Write a complete set of connection equations

\[ -i_A - i_1 - i_3 = 0; \quad i_1 - i_2 = 0; \quad -30 + v_3 = 0; \quad v_1 + v_2 - v_3 = 0; \]

Solve these equations for all currents and voltages

\[ v_A = 30V; \quad v_1 = 10V; \quad v_2 = 20V; \quad v_3 = 30V; \]
\[ i_A = -200mA; \quad i_1 = i_2 = i_3 = 100mA; \]
All you ever wanted to know about resistors …

Resistors are color coded to indicate their value and tolerance.

<table>
<thead>
<tr>
<th>Black</th>
<th>Brown</th>
<th>Red</th>
<th>Orange</th>
<th>Yellow</th>
<th>Green</th>
<th>Blue</th>
<th>Violet</th>
<th>Grey</th>
<th>White</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
</tr>
</tbody>
</table>

This is a 1KΩ resistor with 5% tolerance.

Resistors come at standard values (e.g., at 10% tolerance):

<table>
<thead>
<tr>
<th>.9</th>
<th>1.08</th>
<th>1.35</th>
<th>1.62</th>
<th>1.98</th>
<th>2.43</th>
<th>2.97</th>
<th>3.51</th>
<th>4.23</th>
<th>5.04</th>
<th>6.12</th>
<th>7.38</th>
<th>8.2</th>
<th>Nominal</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.2</td>
<td>1.5</td>
<td>1.8</td>
<td>2.2</td>
<td>2.7</td>
<td>3.3</td>
<td>3.9</td>
<td>4.7</td>
<td>5.6</td>
<td>6.8</td>
<td>8.2</td>
<td>1.08</td>
<td>Lower</td>
</tr>
<tr>
<td>1.1</td>
<td>1.32</td>
<td>1.65</td>
<td>1.98</td>
<td>2.42</td>
<td>2.97</td>
<td>3.63</td>
<td>4.29</td>
<td>5.17</td>
<td>6.16</td>
<td>7.48</td>
<td>9.02</td>
<td>9.02</td>
<td>Upper</td>
</tr>
</tbody>
</table>
Even more than you ever wanted to know about resistors …

The distribution of resistances
Why should this be so?

How can we get a $258.3\,\Omega$ resistor?
Keep testing the ones in the box
A really finely tunable potentiometer
(variable resistor) – a *trimpot*
A coarsely tunable potentiometer in parallel with a lower $R$
Make one from high precision components

Another important characteristic of $R$ is the power rating
Why?